Mechanism for Vanishing Zero Point Energy

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Abstract

In addition to the standard solutions of the quantum field equations having the two forms $e^{\pm(i\omega t - i\mathbf{k}\cdot\mathbf{x})}$, there exist two additional solutions of the form $e^{\pm(i\omega t + i\mathbf{k}\cdot\mathbf{x})}$. By incorporating these latter solutions, deemed "supplemental solutions", into the development of quantum field theory, one finds a simple and natural cancellation of terms that results in an energy VEV, and a cosmological constant, of zero. This fundamental, and previously unrecognized, inherent symmetry in quantum field theory appears to provide a resolution of the large vacuum energy problem, simply and directly, with no modification or extension to the extant mathematics of the theory. In certain scenarios, slight asymmetries could give rise to dark energy.

1 Introduction

1.1 Cosmological Issues

As summarized by Peebles and Ratra[1], Padmanabhan[2], and others, there are presently three overriding cosmologic issues involving phenomena for which no generally accepted theoretical solutions exist: 1) dark matter (non-baryonic, unseen "normal" matter), 2) dark energy (small positive cosmological constant or quintessence), and 3) a vanishing sum of zero-point energies. These may be related, or they may be unrelated. This article focuses on a possible solution to the third of these, though it also suggests a concomitant, potentially viable, approach toward resolving the second.

1.2 Negative Frequencies and Supplemental Solutions

The issue of negative frequency solutions to the relativistic counterparts of the Schroedinger equation has a long and variegated history. Such solutions constitute a second way to solve the quantum field equations beyond those of positive frequency. The question of interpretation of negative frequency solutions, one of the most famous in the history of science, was answered via second quantization. The solutions to the field equations could then be shown to be operators that create and destroy states, rather than being states themselves.

By including the negative frequency solutions, the set of solutions to the quantum field equations was doubled in size. In this article, it is noted that that the number of mathematical solutions to the field equations is actually twice that again. The typically unused half of the full

set is designated herein as the set of "supplemental solutions", and probable reason is suggested for why it has been all but ignored. It is shown that if these supplemental solutions were to generate corresponding vacuum fluctuations, and maintained unbroken their natural symmetry with the traditional solutions, then the theoretical value for vacuum energy density would be zero, and hence, quite close to what is observed.

2 The Unused Solutions to the Field Equations

For simplicity, we consider first the scalar field equation in natural units

$$\left(\partial_\mu \partial^\mu + \mu^2\right) \phi = 0 \tag{1}$$

with traditional eigensolutions

$$e^{-ikx} = e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}, \tag{2}$$

$$e^{ikx} = e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}}. \tag{3}$$

Note, however, that (2) and (3) are not the only forms for solutions to (1). The following, obtained by taking $\omega \to -\omega$ in the above (consider the symbol ω as always a positive number), are also solutions to (1).

$$e^{i\underline{kx}} = e^{i\omega t + i\mathbf{k}\cdot\mathbf{x}} \tag{4}$$

$$e^{-i\underline{kx}} = e^{-i\omega t - i\mathbf{k}\cdot\mathbf{x}} \tag{5}$$

New notation (see underscoring above) and new terminology are herein introduced, wherein (4) and (5) are the "supplemental solutions" to (1). Upon introduction to supplemental solutions, some at first believe them to be already contained in the traditional solutions to the field equations. This issue is addressed in Appendix A

Note that the Dirac and Proça equations have such supplemental solutions as well. The Dirac equation actually has eight eigenspinor solutions, a set of four for $e^{+(i\omega t - i\mathbf{k}\cdot\mathbf{x})}$ and another set of four for $e^{-(i\omega t - i\mathbf{k}\cdot\mathbf{x})}$, as one would expect. Four (two from the first set and two from the second set) have positive energy in the spinor components, and four have negative energy in the spinor components. Tradition (and lack of relevance for real particles) has led to ignoring of the latter four.

In relativistic quantum mechanics[1] (RQM), the precursor to quantum field theory (QFT), (3) presented a problem as it represented a negative energy state. As noted, QFT resolved this by considering solutions having the spacetime dependence of (2) and (3) as operators, not states. Examination of (4) and (5) in the context of RQM leads to a similar issue of negative energy, as well as an additional one. Momentum direction in (4) and (5), if they represent states, is in the opposite direction of wave velocity [see Appendix A], and hence (4) and (5) are unlikely candidates for physical particle states. Probably for this and other reasons, solutions of the form of (4)and (5) fell out of favor early on in the development of RQM.

In parallel with the historical development of (2) and (3) in QFT, however, we can apply second quantization to (4) and (5), determine the resultant field operator solutions, and see if those solutions might provide anything of value in helping to match experiment with theory.

[1] RQM employs the Klein-Gordon, Dirac, and Proça equations, but unlike quantum field theory, the solutions are states, not operators.

3 Supplemental Solutions and QFT

3.1 Symmetry of the Lagrangian

If (4) and (5) solve the same free field equations as (2) and (3), then it follows that the free Lagrangian is symmetric under the transformation $\omega \to -\omega$.

3.2 Klein-Gordon Supplemental Solutions

Quantum field theory formalism for the supplemental solutions can be developed in parallel with the standard approach, using, for scalar fields, the following definitions.

$$\underline{\phi} = \underline{\phi}^{+} + \underline{\phi}^{-} = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k}\right)^{1/2} \left\{\underline{a}(\mathbf{k})e^{i\underline{kx}} + \underline{b}^{\dagger}(\mathbf{k})e^{-i\underline{kx}}\right\} \tag{6}$$

$$\underline{\phi}^{\dagger} = \underline{\phi}^{\dagger+} + \underline{\phi}^{\dagger-} = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k}\right)^{1/2} \left\{\underline{b}(\mathbf{k})e^{i\underline{kx}} + \underline{a}^{\dagger}(\mathbf{k})e^{-i\underline{kx}}\right\} \tag{7}$$

$$\mathcal{L}^{\,0}_{\underline{\phi}} = \partial_\mu \underline{\phi}^{\dagger}\partial^\mu \underline{\phi} - m^2 \underline{\phi}^{\dagger}\underline{\phi}, \tag{8}$$

where (8) is an extra component added to the Lagrangian density for the scalar supplemental solutions. The superscript refers to “free” Lagrangian.

Applying second quantization to the supplemental solutions where

$$\left[\underline{\phi_r}(\mathbf{x},t), \underline{\pi_s}(\mathbf{y},t)\right] = \left[\underline{\phi_r}(\mathbf{x},t), \underline{\dot{\phi}_s}^{\dagger}(\mathbf{y},t)\right] = i\delta_{rs}\delta(\mathbf{x}-\mathbf{y}), \tag{9}$$

using (6), (7) and the definition of the Dirac delta function for finite volume

$$\delta(\mathbf{x}-\mathbf{y}) = \frac{1}{V}\sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}, \tag{10}$$

and equating coefficients of like terms yields the coefficient commutation relations[]

$$\left[\underline{a}(k), \underline{a}^{\dagger}(k')\right] = \left[\underline{b}(k), \underline{b}^{\dagger}(k')\right] = \quad - \quad \delta_{kk'}. \tag{11}$$

Note the above relations differ from their traditional solution counterparts by the minus sign on the RHS[2] [6],[7]. This resulted from the time derivative in (9), since the supplemental solutions have opposite signs from the traditional solutions for the time (energy) term in the exponent.

Using (6) and (7) in the relevant term in the Hamiltonian density

$$\underline{\mathcal{H}}^{0}_{\underline{\phi}} = \sum_r \underline{\pi_r}\underline{\dot{\phi}_r^{\dagger}} - \underline{\mathcal{L}}^{\,0}_{\underline{\phi}} = \underline{\dot{\phi}\dot{\phi}^{\dagger}} + \underline{\dot{\phi}^{\dagger}\dot{\phi}} - \underline{\mathcal{L}}^{\,0}_{\underline{\phi}} = \underline{\dot{\phi}\dot{\phi}^{\dagger}} + \nabla\underline{\phi}^{\dagger}\nabla\underline{\phi} + m^2\underline{\phi}^{\dagger}\underline{\phi} \tag{12}$$

and integrating over all space yields

$$\underline{H} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left\{\underline{a}^{\dagger}(k)\underline{a}(k) \quad - \quad \tfrac{1}{2} \quad + \quad \underline{b}^{\dagger}(k)\underline{b}(k) \quad - \quad \tfrac{1}{2}\right\} \tag{13}$$

[2] Relation (11) leads to an indefinite metric in the Fock space of states and negative norms for some state vectors. See Refs. [3], [4], and [5]. Pauli[6] delineated why operators with such a commutator could not correspond to real particle states. I do not take issue with Pauli on this point. I do suggest that his arguments may not apply to virtual particle states, and that the advantage shown by supplemental solutions in providing a possible answer to the vacuum energy problem was unknown in his time. This issue does, however, need to be resolved.

where for notational streamlining we here and from henceforth drop the sub and superscript notation. Note the $1/2$ terms in (13) have the opposite sign from similar terms in the traditional Hamiltonian

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left\{ a^{\dagger}(k)a(k) \;+\; \tfrac{1}{2} \;+\; b^{\dagger}(k)b(k) \;+\; \tfrac{1}{2} \right\}. \tag{14}$$

The presence of the $1/2$ terms in the traditional development of QFT implied an infinite energy VEV, and necessitated the *ad hoc* introduction of normal ordering in order to keep vacuum energy zero. However, if we consider the total (free, scalar) Hamiltonian as

$$H_{\text{tot}} = H + \underline{H} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left\{ a^{\dagger}(k)a(k) \;+\; b^{\dagger}(k)b(k) \;+\; \underline{a}^{\dagger}(k)\underline{a}(k) \;+\; \underline{b}^{\dagger}(k)\underline{b}(k) \right\} \tag{15}$$

then the $1/2$ terms all drop out and the expectation energy of the vacuum is naturally zero without having to resort to normal ordering[3][8].

3.3 Dirac and Proça Supplemental Solutions

As should be expected, the above analysis has its analogues for spin $1/2$ and spin 1 fields. The Proça equation is so closely related to the Klein-Gordon equation that all results of the preceding sections can be readily extrapolated to spin 1 fields.

It is not quite so obvious, however, that the results of the preceding sections can be extrapolated to Dirac particles. This is because the conjugate momentum for a Dirac field does not involve a time derivative of that field, and hence one cannot expect the Dirac equation solutions to directly parallel (9) through (11). Further, the Dirac Hamiltonian for the traditional solutions has negative terms of $1/2\omega_k$ analogous to the positive such terms in (14).

In Appendix B, the coefficient anticommutaion relations for spin $1/2$ supplemental particles are shown to have the opposite sign from their traditional solutions counterparts, i.e.,

$$\left[\underline{c}_r(p), \underline{c}_s^{\dagger}(p')\right]_+ = \left[\underline{d}_r(p), \underline{d}_s^{\dagger}(p')\right]_+ = \; - \; \delta_{pp'}\delta_{rs}. \tag{16}$$

Using (16) in the free Hamiltonian density for Dirac supplemental particles results in a total Dirac particle Hamiltonian analogous to (15), i.e, having no $-1/2$ terms.

3.4 Supplemental Operators, Propagators, and Observables

Analogous results can be found for supplemental field number operators, propagators, and observables, and derivations of these will be shown in a subsequent article. Number operators yield number eigenvalues of opposite sign from their traditional counterparts. Supplemental propagators have the same form, but opposite sign from traditional propagators. Observables of states created and destroyed by supplemental operators, in general, do not correspond to those of the physically manifest world.

[3] Teller (Ref. [8]) discusses aspects of QFT that seem unnatural and inelegant, and he includes normal ordering of the Hamiltonian as one of these. He submits that a complete and true theory would not have such an artificial, and arbitrarily imposed, feature. On page 130, with reference to normal ordering he states, "If, as appears to be the case, at this point one must use mathematically illegitimate tricks, concern is an appropriate response." It is noteworthy that nowhere else in QFT are we permitted to simply assume that non-commuting operators temporarily commute. This seeming contradiction is resolved by incorporating supplemental solutions into the theory.

Of particular note, the total Hamiltonian expectation value for a supplemental particle state is negative. Further, the total three-momentum expectation value for a supplemental particle state is in the opposite direction of particle velocity. Such characteristics are not those of real particles, though they can be so for virtual particles. For example, the virtual exchange between two oppositely charged macroscopic bodies must entail three-momentum in the opposite direction of travel of the virtual particles in order for the bodies to attract. And virtual loop diagrams are integrated over both positive and negative energies for the individual virtual particles therein. Further, scalar (timelike polarization) virtual photons have negative energies.[9] Still further, fermion zero-point energies, as noted above, are negative.

3.5 Nature of Supplemental Particles

Hence, we postulate herein that if supplemental states are indeed realized in the spectrum of states, they are necessarily constrained to be virtual, cannot be real, and are never directly observed. No symmetry breaking mechanism (at least none much above contemporary energy levels, such as that conjectured between particles and sparticles in supersymmetry) is envisioned between the traditional and supplemental particles. And to be clear, the supplemental particles, though having negative energy states, are not a reincarnation of Dirac's "sea of negative energy", but quite a different thing entirely.

4 Vacuum Energy and the Cosmological Constant

4.1 Cancellation of Zero-point Energy Fluctuations

Weinberg[10], among others, notes that summing of the zero-point energies ($1/2$ quantum at each frequency) of all normal modes of some field of mass m up to a wave number cutoff $k_c \gg m$ yields a vacuum energy density (with $\hbar = c = 1$)

$$< \rho_0 > = \int_0^{k_c} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \; \cong \; \frac{k_c^4}{16\pi^2}. \tag{17}$$

For a suitable cutoff on the order of the Planck scale, we get

$$< \rho_0 > \cong 2 \times 10^{71} \text{ GeV}^4, \tag{18}$$

which, being off from the observed density of the vacuum

$$\rho_V \; \leq \; 10^{-47} \text{ GeV}^4 \tag{19}$$

by a factor on the order of 118 decimal places, is the well-known biggest discrepancy between theory and experiment in the history of science.

Further, the zero point energy density from $1/2$ quanta normal modes $\langle \rho_0 \rangle$ predicted by traditional quantum field theory does not result in pressures $\langle p_0 \rangle = -\langle \rho_0 \rangle$. (See background review in Appendix C.) This implies that vacuum fluctuations cannot manifest as a cosmological constant, regardless of size. More importantly, the zero point stress-energy tensor from $1/2$ quanta normal modes $\langle T_0^{\mu\nu} \rangle$ is then not Lorentz invariant. Further, in an expanding universe, vacuum energy density would not be constant.

Consider, however, that inclusion of the supplemental solutions into the theory results in a complete cancellation of each positive energy vacuum normal mode by a negative energy vacuum normal mode of the precise same magnitude, and hence a null total zero point energy density VEV, i.e.,

$$<_{Total}\ \rho_0> = <\rho_0 + \underline{\rho_0}> = 0, \tag{20}$$

which is consistent with (19) and constant over time. The stress-energy tensor $\langle_{Total}T_0^{\mu\nu}\rangle$ is also null, and thus obviously Lorentz invariant.

As discussed by Weinberg, Ellwenger[11], Peebles and Ratra[1], and others, some mechanisms investigated to null out the zero point energy, such as supersymmetry, work only to certain energy scales, resulting in enormous predicted values for our epoch. Only a mechanism that transcends symmetry breaking scales and is effective over virtually *all* energy levels can possibly produce a null (or near null) ρ_V. Supplemental solutions provide such a scale invariant symmetric mechanism.

4.2 Scalar Field Vacuum Potentials

Higgs, inflaton, and quintessence theories posit scalar fields with potentials leading to a post symmetry breaking Lagrangian density for a real field of the form[12],[13],[14]

$$\mathcal{L}_\phi = \tfrac{1}{2}\left(\partial_\mu\phi g^{\mu\nu}\partial_\nu\phi - m^2\phi^2\right) - V, \tag{21}$$

where V is constant on the order of m^4. This leads to mass-energy density and pressure in local Minkowski coordinates of

$$T_\phi^{00} = \rho_\phi = \tfrac{1}{2}\left(\left(\dot{\phi}\right)^2 + (\nabla\phi)^2 + m^2\phi^2\right) + V, \tag{22}$$

$$T_\phi^{11} = p_\phi = (\partial_1\phi)^2 + \tfrac{1}{2}\left(\left(\dot{\phi}\right)^2 - (\nabla\phi)^2 - m^2\phi^2\right) - V. \tag{23}$$

Einstein's field equations are

$$G^{\mu\nu} - \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu} + 8\pi G\left\langle T^{\mu\nu}\right\rangle. \tag{24}$$

where Λ is the classical cosmological constant. From the results of Appendix C, (22), and (23), one can see that

$$T_\phi^{00} = \frac{1}{V_s}\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left\{a^\dagger(k)a(k) \quad + \quad \tfrac{1}{2}\right\} + V \tag{25}$$

$$T_\phi^{11} = \frac{1}{V_s}\sum_{\mathbf{k}}\frac{(k_1)^2}{\omega_{\mathbf{k}}}\left\{a^\dagger(k)a(k) \quad + \quad \tfrac{1}{2}\right\} - V, \tag{26}$$

where V_s is spatial volume. Thus, the vacuum expectation values become

$$\left\langle T_\phi^{00}\right\rangle = \langle\rho_\phi\rangle = \frac{1}{V_s}\sum_{\mathbf{k}}\frac{\omega_{\mathbf{k}}}{2} + V \qquad \left\langle T_\phi^{11}\right\rangle = \langle p_\phi\rangle = \frac{1}{V_s}\sum_{\mathbf{k}}\frac{(k_1)^2}{2\omega_{\mathbf{k}}} - V. \tag{27}$$

If one ignores the summations, then from (24), the constant potential V acts like a contribution to the effective cosmological constant

$$\Lambda_{\text{eff}} = \Lambda + \Lambda_\phi = \Lambda + 8\pi G\left\langle\rho_\phi\right\rangle = \Lambda + 8\pi G V \tag{28}$$

(or alternatively, like the vacuum density of the scalar field $\langle \rho_\phi \rangle = -\langle p_\phi \rangle$, where pressure is negative.)

Two significant problems exist with this approach.

1. The calculated order of Λ_ϕ differs from observation by many orders of magnitude. For electroweak symmetry breaking, V is about 10^8 GeV^4. (Compare with (19)) For GUT symmetry breaking, V is on the order of 10^{64} GeV^4.

2. There is no justification for ignoring the ½ quanta normal vacuum modes summation in the VEV's of (27), which as noted earlier, are not only immense, but destroy vacuum Lorentz invariance and constancy.

However, if supplemental particles $\underline{\phi}$ are included in the calculations, the vacuum expectation stress energy tensor should be null. The vacuum fluctuation summations then have two components, equal in magnitude and opposite in sign, that cancel. Additionally, given the form for the supplemental particle Hamiltonian (13), it is reasonable to expect the supplemental scalar field ϕ Lagrangian to have, due to symmetry, the similar functional dependence as (21), but with opposite sign for the constant potential energy density[4] [15],[16], i.e.,

$$\underline{\mathcal{L}_\phi} = \tfrac{1}{2}\left(\partial_\mu \underline{\phi} g^{\mu\nu} \partial_\nu \underline{\phi} - m^2 \underline{\phi}^2\right) - \underline{V} = \tfrac{1}{2}\left(\partial_\mu \underline{\phi} g^{\mu\nu} \partial_\nu \underline{\phi} - m^2 \underline{\phi}^2\right) + V. \tag{29}$$

With this one gets

$$\underline{T_\phi^{00}} = \underline{\rho_\phi} = \tfrac{1}{2}\left(\left(\dot{\underline{\phi}}\right)^2 + \left(\nabla \underline{\phi}\right)^2 + m^2 \underline{\phi}^2\right) - V \tag{30}$$

$$\underline{T_\phi^{11}} = \underline{p_\phi} = \left(\partial_1 \underline{\phi}\right)^2 + \tfrac{1}{2}\left(\left(\dot{\underline{\phi}}\right)^2 - \left(\nabla \underline{\phi}\right)^2 - m^2 \underline{\phi}^2\right) + V, \tag{31}$$

and

$$\left\langle {}_{Tot}T_\phi^{00} \right\rangle = \left\langle \rho_\phi + \underline{\rho_\phi} \right\rangle = \underbrace{\sum_{\mathbf{k}} \tfrac{1}{2V_s}\left(\omega_{\mathbf{k}} - \omega_{\mathbf{k}}\right)}_{\text{from zero point fluctuations}} + \underbrace{V - V}_{\text{from scalar potentials}} = 0 \tag{32}$$

$$\left\langle {}_{Tot}T_\phi^{11} \right\rangle = \left\langle p_\phi + \underline{p_\phi} \right\rangle = \underbrace{\sum_{\mathbf{k}} \tfrac{1}{2V_s}\left(\tfrac{(k_1)^2 - (k_1)^2}{\omega_{\mathbf{k}}}\right)}_{\text{from zero point fluctuations}} - \underbrace{V + V}_{\text{from scalar potentials}} = 0. \tag{33}$$

The zero point fluctuation summations from the supplemental and traditional particles, as well as the post symmetry breaking level potentials, cancel, leaving a Lorentz invariant, constant, null stress-energy tensor for the vacuum and no cosmological constant.

[4] Linde (Refs [15], [16]) also suggests negative energy particles in order to obtain a null cosmological constant, but, unlike the present approach, starts by augmenting the Lagrangian with a term equal to the original Lagrangian but having opposite sign. Linde's approach and the one shown herein have some similarities, but are quite different in other regards. One of the similarities is the positing of an additional scalar potential with opposite sign, such as suggested in this section. Certain possible results from adopting this approach that are discussed in this and the subsequent two sections were also noted by Linde

4.3 Small Cosmological Constant

Consider a slight asymmetry between one scalar field and the other with regard to their constant vacuum potentials, wherein

$$\underline{\mathcal{L}}_{\underline{\phi}} = \tfrac{1}{2}\left(\partial_\mu \underline{\phi} g^{\mu\nu} \partial_\nu \underline{\phi} - m^2 \underline{\phi}^2\right) \; + \; \underline{V}, \tag{34}$$

such that

$$V = \underline{V} + \delta, \tag{35}$$

with δ small. Using this in (32) and (33), we would have

$$\left\langle {}_{Tot}T^{\mu\nu}_{\phi} \right\rangle = \delta \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \tag{36}$$

yielding a small cosmological constant, as well as a Lorentz invariant stress-energy tensor for the vacuum with constant vacuum energy density.

This is as we see it today, although we still need a large cosmological constant in earlier epochs for inflation to be viable.

4.4 Inflation

Possibilities exist for time dependent δ. A large δ at earlier times that relaxed to a present epoch smaller value would be one way to satisfy inflationary requirements.

In inflation scenarios (prior to, and concurrent with, a symmetry breaking process), the scalar field Lagrangian density is typically expressed as

$$\mathcal{L}_\phi = \tfrac{1}{2}\partial_\mu \phi g^{\mu\nu} \partial_\nu \phi \; - \; V(\phi) \tag{37}$$

where $V(\phi)$ can have a ϕ dependence as depicted in the top half of Figure 1. Inflation begins when the energy density of the ϕ field is concentrated in a high, relatively flat potential (the false vacuum at time t_1 in the figure) and ends with the ϕ field in its lowest potential state (the true vacuum at time t_2). The remnant value V then becomes part of the post symmetry breaking Lagrangian (see (21)[5]).

Consider, on the bottom part of Figure 1, a supplemental particle potential, which could be expected to be a mirror image of the traditional potential of the left side. The particles we see in our universe, including all those coupled to the ϕ field, would have their destinies tied to the potential $V(\phi)$, and presumably not to $\underline{V}(\underline{\phi})$. Yet, the vacuum, and its properties, would be tied to both (the sum of the two.)

A range of possibilities exist with regard to the temporal relationship between the two sides of Figure 1. Consider one extreme in which both potentials (as they appear in the figure) are precise mirrors of each other, and both the ϕ and $\underline{\phi}$ fields move along their respective potential curves in lockstep. At each moment in time, the expectation values of each field are located at

[5] The ϕ field in (21) is different from the ϕ field in (37), however. The former represents a “new” particle type whose zero real density state corresponds to the true vacuum of time t_2 in Figure 1. The latter represents a different (though closely related) particle whose zero real density state corresponds to the false vacuum of time t_1 in Figure 1.

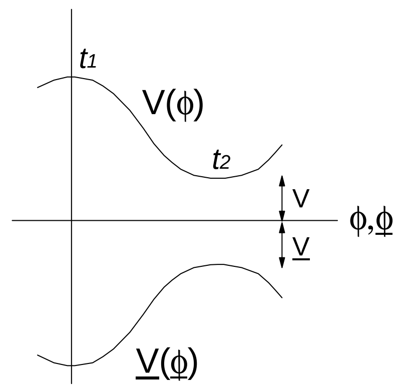

Figure 1: . Possible Traditional and Supplemental Scalar Field Potentials

the same point along their respective horizontal axes. At no time would we have an inflationary occurrence, since the total vacuum potential, and thus the vacuum stress-energy tensor, would always be zero.

To get inflation, at least one of two possibilities need exist. First, the potentials might, for some reason, not be precise mirror images of one another. This could result in a large total potential (due to the sum of the two field potentials) at early times, but a very small total potential in the present (giving rise to a small cosmological constant.) This would, of course, also give rise to different time evolutions for the expectation values of the two fields.

Second, the time evolution along the two curves might differ, even if they have the same (but inverted) shapes. The supplemental potential vacuum, mirroring its traditional counterpart, would tend to seek the state of *highest* potential (lowest magnitude potential). With the real particles of the universe having initial high positive energy, perhaps the impetus for the supplemental field to seek its (inverted) minimum would be stronger than that of the traditional field. Thus, the supplemental field might reach its expectation value for the true vacuum while the traditional field was still at the false vacuum. The net potential from the sum of the two potentials at that time would have a large positive value and initiate inflation, which would continue until the traditional field reached true vacuum. If any supplemental particles exist at that time, during inflation they would be thinned out to effectively zero density, leaving a net positive energy particle density in the universe.

A small residual potential might exist if the last part of the journey takes an extended time, so a small discrepancy in magnitude difference between the two fields still remains. In time, this residual could be expected to diminish to zero.

5 Antipodal Symmetry and Supplemental Solutions

Linde[16] conjectured "some mechanism, associated probably with some kind of symmetry of the elementary particle theory, which would automatically lead to a vanishing of the cosmological constant in a wide class of theories". He considered the possibility of two quasi-independent "universes" with the same particle equations of motion, but with opposite signs for the Lagrangian, and thus for energy, for which he coined the term "antipodal symmetry". One universe (ours) has positive energy particles. The other is a mirror image with negative energy particles. The total energy, and the total cosmological constant, net to zero.

As shown herein, such a symmetry already exists within the mathematics of quantum field theory. Though largely unrecognized, mathematical solutions of the extant theory provide the desired symmetry. Nothing more is needed. The solution is direct, complete, and remarkably simple. The fundamental question then is whether or not nature in her physical manifestation parallels the nature of her mathematics. Do virtual supplemental particles actually exist?

A partial answer, as noted in Section 3.4, is that traditional QFT calculations already encompass (half of the time) virtual particles with negative energy and other characteristics of supplemental particles. It is not a big step to consider vacuum fluctuations and scalar field potentials following suit.

The absence of observed real supplemental particles may be conjectured as being related to the fact that reversing the arrow of time in the supplemental solutions produces the traditional solutions. That may mean, with regard to perception, that real supplemental particles travel backward in time. Thus one might speculate that the universe needed no initial energy from which to begin, with equal amounts of traditional and supplemental particles emerging from nothing. From there, the real negative energy supplemental particles traveled backward in time, and in the process created their own universe, which would appear to any beings in that universe to possess positive energy. The traditional particles, traveling forward in time, created our universe and appear to us as having positive energy. Each universe would have only one kind of real particle, but virtual particles of both types. And this would result in null vacuum energy and null (or near null) cosmological constants in both[6].

In this scenario, the act of creation itself would have been the first, and most fundamental, symmetry breaking of nature's laws.

6 Summary

A previously unrecognized, but fundamental, symmetry in elementary particle theory exists in which supplemental solutions to the QFT field equations are obtained from the traditional solutions by taking $\omega \to -\omega$. The implementation of this symmetry in the theory results in a total Hamiltonian yielding a null energy VEV, renders the *ad hoc* procedure of normal ordering unnecessary, and predicts a null cosmological constant. An asymmetry between the traditional and supplemental particle solutions could lead to a non-null cosmological constant, whose magnitude and evolution would depend on the nature and degree of the asymmetry. Agreement with observations is only maintained if supplemental states occur only as virtual, and not as real, particles, and given the properties of the supplemental states, this appears reasonable.

The supplemental/traditional solution symmetry can be maintained over all energy scales, unlike other attempts to null out vacuum energy, such as supersymmetry, which only succeed down to particular, non-contemporary, energy levels. This symmetry provides a simple, direct solution to the predicted enormous vacuum energy problem, and that solution is already built into the original theory. No new theory, and no *ad hoc* complications are required.

[6] Linde[16] cautions that ".. it is dangerous to consider theories containing particles *with both signs of energy*, which typically occurs in the theories containing particles with indefinite metric.." but notes that "..in our case no such problem does exist [because] the [traditional] fields do not interact with the [negative energy] fields.." In the present suggested scenario, real traditional particles would not interact with real supplemental particles, because the latter would not be around. Alternatively, the traditional and supplemental particles could be completely decoupled, yet each would affect the properties of spacetime in opposite ways.

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8 Appendix A: Supplemental Solutions Independence from the Traditional Solutions

Since the traditional solutions of the Klein-Gordon equation

$$\phi = \phi^{+} + \phi^{-} = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k}\right)^{1/2} \left\{a(\mathbf{k})e^{ikx} + b^{\dagger}(\mathbf{k})e^{-ikx}\right\} \tag{38}$$

are summed over all $\mathbf{k}$, for every $k_x = 10$, for example, in (38), a $k_x = –10$ is also summed. Thus, some might reason, for each $+E$ in the summation with a $–k_x$, there is a $+E$ with with a $+k_x$, and similarly for $–E$, leading to the solution forms (4) and (5). There are, however, subtle differences between sign in an algebraic relation and sign of an algebraic quantity, which is illustrated with two elementary examples as follows.

Example 1

Consider the difference between sign in an algebraic expression and sign of an algebraic quantity in that expression. For example,

$$x - y = 0 \tag{39}$$

has fully symmetric values for y. For every positive value of y, there is a negative value of same magnitude. But that does not mean that the relation

$$x + y = 0 \tag{40}$$

is contained within (39). Precisely parallel logic applies to supplemental vs traditional solutions.

Example 2

For ease of illustration, consider the K-G solutions as states, as in relativistic quantum mechanics. Take the expression for constant phase for a wave traveling along the x axis

$$\omega t - k_x x = \text{ constant,} \tag{41}$$

differentiate with respect to t, and solve for the wave velocity.

$$v \ = \ dx/dt \ = \ \omega/k_x \tag{42}$$

and velocity has the same sign as k_x (momentum and velocity are in same direction.) For k_x having opposite sign, v also has opposite sign.

Repeat for the supplemental solutions.

$$\omega t + k_x x = \text{ constant} \tag{43}$$

and

$$v \ = \ dx/dt \ = \ \text{-}\omega/k_x. \tag{44}$$

So whatever the sign on the 3 momentum k_x, the velocity of the wave is in the opposite direction. Change the sign of k_x, and the velocity is still in the opposite direction of k_x (it changes sign too).

While it may seem bizarre to have velocity and 3-momentum in opposite directions, there is no doubt that it is a radically different result, with radically different physical implications. In fact, it is so different that no real, physical particles/waves display it. When summing over all positive and negative values of k_x in the traditional general solution for ϕ, one does NOT get this result for the negative k_x values (velocity is still in the direction of k_x for negative k_x).

As noted in Section 3.4, the same thing is true in quantum field theory, where the solutions are operator fields, rather than states.

9 Appendix B: Supplemental Dirac Anti-commutators

To derive the coefficient anti-commutator relations of (16), begin with the Dirac equation for one of the supplemental particle solutions,

$$(i\gamma^\mu \partial_\mu - m)\underline{\psi}^+ = 0. \tag{45}$$

Insert

$$\underline{\psi}^+ = \underline{u}(\mathbf{p})e^{\underline{ipx}}, \tag{46}$$

choose a representation for gamma matrices such as the standard rep, and solve the resulting eignenvalue problem for $\underline{u}^{(i)}$, where i= eigenvector number (1,2,3,4). Each eigenvector has four (implicit and unlabeled here) components in the representation space.

If, in parallel with the traditional approach, one then defines a normalized vector $\underline{u}_r$ as

$$\underline{u}_r = (\tfrac{-|E|+m}{2m})^{1/2}\underline{u}^{(r)}, \tag{47}$$

we find

$$\underline{u}_r^\dagger(\mathbf{p})\underline{u}_s(\mathbf{p}) = \tfrac{-|E|}{m}\delta_{rs}, \tag{48}$$

which has the opposite sign on the RHS from the similar traditional relationship. Repeating the procedure for $\underline{\psi}^-$ yields the same relation for $\underline{v}_r(\mathbf{p})$.

Normalizing (46) according to (47), applying second quantization (with anti-commuting fields) to the supplemental Dirac solutions, and using (48) in parallel fashion to steps (9) through (11), one ends up as promised with (16), the coefficient anti-commutation relations for supplemental Dirac particles.

10 Appendix C: Real Scalar Field Vacuum Stress Energy Tensor

From the free real scalar field Lagrangian density

$$\mathcal{L}_\phi = \tfrac{1}{2}\left(\partial_\mu\phi g^{\mu\nu}\partial_\nu\phi - m^2\phi^2\right) \tag{49}$$

it can be shown[17] that

$$T^{00}_\phi = \rho_\phi = \tfrac{1}{2}\left(\left(\dot{\phi}\right)^2 + (\nabla\phi)^2 + m^2\phi^2\right) \tag{50}$$

$$T_\phi^{11} = p_\phi = (\partial_1 \phi)^2 + \tfrac{1}{2}\left(\left(\dot{\phi}\right)^2 - (\nabla\phi)^2 - m^2\phi^2\right). \tag{51}$$

The real scalar field and its derivatives are

$$\phi = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V_s\omega_{\mathbf{k}}}}\left(a(\mathbf{k})e^{-ikx} + a^\dagger(\mathbf{k})e^{ikx}\right) \tag{52}$$

$$\dot{\phi} = \sum_{\mathbf{k}} \frac{i\omega_{\mathbf{k}}}{\sqrt{2V_s\omega_{\mathbf{k}}}}\left(-a(\mathbf{k})e^{-ikx} + a^\dagger(\mathbf{k})e^{ikx}\right) \tag{53}$$

$$\phi_{,i} = \sum_{\mathbf{k}} \frac{ik_i}{\sqrt{2V_s\omega_{\mathbf{k}}}}\left(a(\mathbf{k})e^{-ikx} - a^\dagger(\mathbf{k})e^{ikx}\right), \tag{54}$$

where V_s is spatial volume. The first term on the RH of (50) is

$$\tfrac{1}{2}\dot{\phi}\dot{\phi} = \tfrac{1}{2}\left(\sum_{\mathbf{k}} \tfrac{i\omega_{\mathbf{k}}}{\sqrt{2V_s\omega_{\mathbf{k}}}}\left(-a(\mathbf{k})e^{-ikx} + a^\dagger(\mathbf{k})e^{ikx}\right)\right)\left(\sum_{\mathbf{k}'} \tfrac{i\omega_{\mathbf{k}'}}{\sqrt{2V_s\omega_{\mathbf{k}'}}}\left(-a(\mathbf{k}')e^{-ik'x} + a^\dagger(\mathbf{k}')e^{ik'x}\right)\right). \tag{55}$$

In

$$\langle \phi_{\mathbf{k}} | \rho_\phi | \phi_{\mathbf{k}} \rangle \tag{56}$$

where ρ_ϕ is an operator represented by (50), all terms with $\mathbf{k} \neq \mathbf{k}'$, drop out, as do all terms in $a(\mathbf{k})a(\mathbf{k})$ and $a^\dagger(\mathbf{k})a^\dagger(\mathbf{k})$. (55) is part of (50), so that part reduces to

$$\tfrac{1}{2}\dot{\phi}\dot{\phi} \rightarrow \tfrac{1}{2}\sum_{\mathbf{k}} \tfrac{(\omega_{\mathbf{k}})^2}{2V_s\omega_{\mathbf{k}}}\left(a(\mathbf{k})a^\dagger(\mathbf{k}) + a^\dagger(\mathbf{k})a(\mathbf{k})\right) = \tfrac{1}{2V_s}\sum_{\mathbf{k}} \omega_{\mathbf{k}}\left(a^\dagger(\mathbf{k})a(\mathbf{k}) + \tfrac{1}{2}\right), \tag{57}$$

where we have used the commutation relations in the last step. Similarly,

$$\tfrac{1}{2}(\nabla\phi)^2 \rightarrow \tfrac{1}{2V_s}\sum_{\mathbf{k}} \tfrac{|\mathbf{k}|^2}{\omega_{\mathbf{k}}}\left(a^\dagger(\mathbf{k})a(\mathbf{k}) + \tfrac{1}{2}\right) \tag{58}$$

$$\tfrac{1}{2}m^2\phi^2 \rightarrow \tfrac{1}{2V_s}\sum_{\mathbf{k}} \tfrac{m^2}{\omega_{\mathbf{k}}}\left(a^\dagger(\mathbf{k})a(\mathbf{k}) + \tfrac{1}{2}\right). \tag{59}$$

Since $(\omega_{\mathbf{k}})^2 = m^2 + |\mathbf{k}|^2$, the above three relations summed reduce to the well-known operator form for (50)

$$\rho_\phi = \frac{1}{V_s}\sum_{\mathbf{k}} \omega_{\mathbf{k}}\left(a^\dagger(\mathbf{k})a(\mathbf{k}) + \tfrac{1}{2}\right), \tag{60}$$

which has the non-zero VEV $< \rho_\phi >$ equal to the sum of the $\omega_k/2$ terms.

In similar fashion, the pressure operator of (51) reduces to

$$p_{1\phi} = \frac{1}{V_s}\sum_{\mathbf{k}} \frac{|k_1|^2}{\omega_{\mathbf{k}}}\left(a^\dagger(\mathbf{k})a(\mathbf{k}) + \tfrac{1}{2}\right) \tag{61}$$

and therefore from (60) and (61)

$$\langle \rho_\phi \rangle \neq -\langle p_{1\phi} \rangle. \tag{62}$$

Thus, the vacuum fluctuations contribution to the stress energy tensor cannot result in a cosmological constant, which must have the form (in local Minkowskian coordinates)

$$\Lambda_V \eta^{\mu\nu} = 8\pi G \begin{bmatrix} \langle\rho_\phi\rangle & 0 & 0 & 0 \\ 0 & -\langle\rho_\phi\rangle & 0 & 0 \\ 0 & 0 & -\langle\rho_\phi\rangle & 0 \\ 0 & 0 & 0 & -\langle\rho_\phi\rangle \end{bmatrix}. \tag{63}$$

Neither can the vacuum stress-energy tensor be Lorentz invariant, as only a tensor of the form $C\eta^{\mu\nu}$, where C is a constant, can be so.

It is well known[18], and derivable from the Einstein field equations with a Friedmann metric, that the rate of change of density in the universe is

$$\dot{\rho} = -3\left(\rho + p\right)\frac{\dot{a}}{a}, \tag{64}$$

where a is the time dependent scale factor of the universe (the "radius" of the universe). So from (64), the mass-energy density of the vacuum in an expanding universe can only remain constant if the vacuum pressure is equal in magnitude and opposite in sign from the vacuum density. As shown above in (62), this is not the case for traditional fields vacuum fluctuation energy density.

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